

# Practical Aspects of Rogowski Current Transducer Performance

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## Abstract

This paper examines the frequency response for a 10MHz bandwidth Rogowski transducer. The transducer was tested with currents from a tuned LC circuit for frequencies between 0.6 and 13MHz in comparison with a 20MHz coaxial shunt. The sensitivity variation with frequency compares not unreasonably with theoretical predictions. The response was also examined at the transition frequency from active to passive integration and the phase displacement shown to be typically 0.1 degree.

## 1. Introduction

Over the past decade Rogowski current transducers have become widely used as a convenient clip-around instrument particularly for applications with large currents or where high bandwidth measurements are necessary. Figure 1. shows a typical transducer with a relatively thin flexible Rogowski coil connected to a battery powered integrator.



Figure 1. The **CWT** Rogowski current transducer

Following earlier publications [1 to 5] defining the basic principles of this measurement technique, in 1999 the authors presented papers [6,7] outlining the factors influencing the behaviour of Rogowski transducers used for measuring currents with very high frequency components such as in power semiconductor switches. In particular these papers showed that undesirable measurement effects, such as pre-shoot and ringing, could be avoided by using a non-inverting integrator. Since then the high frequency bandwidth of Rogowski transducers used as a general purpose instrument has increased to 10MHz [8] whilst retaining a low frequency bandwidth of typically 1Hz.

Whilst [6-8] gave theoretical predictions of high frequency performance and showed current pulse measurements which demonstrated the high bandwidth, they did not provide a gain/phase versus frequency relationship based on practical measurements. This was mainly due to the lack of MHz current sources of sufficiently large current amplitude but this has now been overcome and the major aim of the paper is to present practical gain / phase measurements at high frequencies for comparison with theoretical prediction.

Ideally the transducer has a constant sensitivity  $R_{sh}$  (e.g. 2mV/A) with zero phase error over the frequency range between its upper and lower bandwidth limits although variation will occur as the bandwidth limits are approached. However, the non-inverting integrator utilises [9] active (op-amp) integration at low frequencies and passive (CR) integration at high frequencies. Inexact matching of the respective integration time constants will result in different gains at high and low frequencies and phase error around the transition frequency. For example a 1% difference in time constants will result in a 1% gain difference and a phase error at the transition frequency of 0.3 degrees. A further aim of the paper is to discuss how the

matching is achieved and to show from practical measurements that the error is typically only 0.1 degrees.

## 2. Equivalent Circuit and Theoretical Behaviour

The theoretical behaviour of the Rogowski transducer has been well publicised [1,..8], nevertheless it is helpful for the purpose of this paper to review this and in particular the aspects particularly affecting high frequency performance.

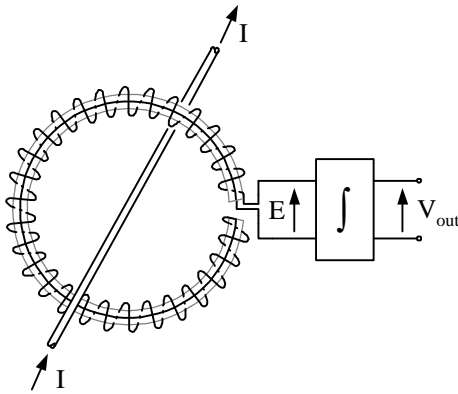


Figure 2. The basic Rogowski transducer

The Rogowski coil, illustrated by Figure 2, comprises a plastic former with a winding of uniform turns density  $N$  (turns/m) and turn area  $A$  ( $m^2$ ). If it is formed into a closed loop surrounding the current to be measured  $I$  then the voltage  $E$  induced in the coil is theoretically independent of the loop shape or the position of current within the loop and is given by

$$E = H \frac{dl}{dt} \quad (1)$$

where the coil sensitivity  $H$  (Vs/A) is given by

$$H = \mu_0 NA \quad (2)$$

Figure 3 shows the basic equivalent circuit for a Rogowski coil connected to an integrator utilising a non-inverting high bandwidth low noise op-amp circuit which [6,7] gives better high frequency performance than the conventional inverting circuit.

At most frequencies the Rogowski coil can be simply represented [4] by its distributed

inductance  $L$  and effective capacitance  $C'$  as lumped parameters shown in Figure 3. However, at very high frequencies approaching the bandwidth of the coil ( $1/2\pi\sqrt{LC'}$ ) the dynamics performance of the coil becomes dependant on current position. This has been discussed in reference [8] and is further commented upon below.

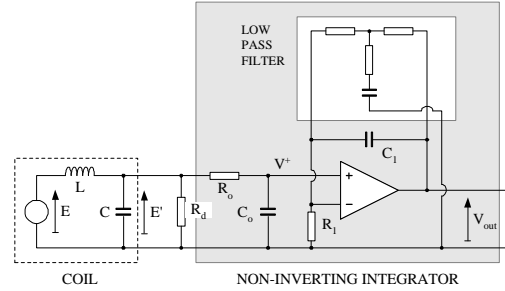


Figure 3. Transducer with non-inverting integrator

To avoid ripples or reflections in the coil a damping resistor  $R_d$  is fitted at the coil termination, together with a series resistor  $R_o$  which forms part of a passive integrator network. The capacitor part  $C_o$  of this network includes the capacitance of the co-axial cable connecting the coil to the integrator unit. At very high frequencies the cable is not purely capacitive and the behaviour is modified from the lumped parameter  $R_o C_o$  representation shown in Figure 3. It may be shown that the modified effect gives a slight increase in gain and a slight phase lead (typically 5% and 5 degrees at the bandwidth frequency).

At very high frequencies the integration is achieved by the passive network  $C_o R_o$  and the op-amp circuit approximately behaves a unity gain amplifier with a simple time delay  $T_b$  equal to  $1/(2\pi GB)$  where  $GB$  is the op-amp gain bandwidth product. The overall transfer function representing high frequency behaviour is therefore.

$$\frac{V_{out}}{I} = R_{sh}(\omega) = \frac{R_{sho} \cdot F_1(\theta) \cdot F_2(\omega) \cdot e^{-j\theta_a}}{[1 + j2\zeta_c \omega T_c - (\omega T_c)^2][1 + j\omega T_b]} \quad (3)$$

where

- $R_{sho}$  = nominal sensitivity at hf =  $H / C_o R_o$
- $\theta_a$  = delay angle for cable
- $\zeta_c$  = effective damping ratio for coil

$T_c$  = coil delay  $\sqrt{LC}$   
 $T_b$  = op-amp delay  
 $F_1(\theta)$  = correction for current position at high frequency  
 $F_2(\omega)$  = correction for cable effect at high frequency  
 $\ddot{e}$  =  $0.5\ddot{\theta}T_c$

For currents close to the coil but within the loop  $F_1(\theta)$  is approximately defined [8] as

$$F_1(\theta) = \frac{1}{2} \left\{ 1 + \frac{\theta \cos(\lambda\theta)}{\sin \theta} \right\} \quad (4)$$

where  $\ddot{e}$  is the per unit distance of the current position from the free end of the coil.

The background of equation (3) has been discussed in detail in [8] and it is inappropriate to repeat this in this paper except to say that the coil-cable-integrator behaviour at very high frequencies is very complex and (3) is only a good approximation.

At low frequencies the passive integrator  $C_oR_o$  merely acts as unity gain and the integration is achieved by the op-amp circuit with an integration time constant  $C_1R_1$ . The nominal low frequency sensitivity is therefore  $R_{sh1} = H/C_1R_1$  and for consistency with the high frequency sensitivity  $C_1R_1$  requires accurate tuning when calibrating the transducer such that  $C_1R_1 = C_oR_o$ . Calibration difficulty is a disadvantage of the non-inverting integrator, although not as great as for the integration method of reference [3] for which three time constants require accurate matching.

At very low frequencies, around and below the low frequency bandwidth limit, the integrator behaviour is modified by the low pass filter [5] shown in Figure 3. to give a 40dB/decade roll off in sensitivity according to the overall sensitivity relationship

$$\frac{V_{out}}{I} = \frac{R_{sh1}(j\omega T)^2}{(1 + j2\zeta\omega T - (\omega T)^2)} \quad (5)$$

where

$R_{sh1}$  = nominal sensitivity at low frequency  
 $T$  (typically 0.2 to 1.0 secs) is the filter time constant.

At intermediate frequencies, spanning the range of transition from active to passive integration, the transducer sensitivity is given by [6,7,8]

$$\frac{V_{out}}{I} = \frac{R_{sh1}(1 + j\omega T_i)}{(1 + j\omega T_o)} \quad (6)$$

where  $T_o = C_oR_o$  and  $T_i = C_1R_1$ .

If the time constants are not accurately matched there will be a disparity in sensitivity ( $R_{sho} = (T_i/T_o) \times R_{sh1}$ ) and a phase error  $\phi_e$  given by

$$\phi_e = \tan^{-1}(\omega T_i) - \tan^{-1}(\omega T_o) \quad (7)$$

This error will be a maximum at the transition frequency  $\omega = 1/\sqrt{T_i T_o}$  given by

$$\phi_{e \max} = \tan^{-1} \left\{ \frac{0.5(T_i - T_o)}{\sqrt{T_i T_o}} \right\} \quad (8)$$

In practice this error is very small (< 1 degree) due to good matching of  $T_i$  and  $T_o$ .

### 3. Calibration and Matching

In most cases the transducers are calibrated [10] at 50Hz using a current of appropriate magnitude (e.g. 2000A rms for a 2mV/A transducer) controlled to within 0.1%. The output is compared using a DVM with that of a NAMAS Calibrated CT and the active integrator time constant is set by adjusting  $R_1$  to give the specified transducer sensitivity.

The passive integrator time constant is then set using a 15 $\mu$ s current pulse of appropriate magnitude (e.g. 1400A peak) with fast rising and falling edges. The output is compared with that of a 20MHz Pearson CT and  $C_o$  is adjusted to match the waveforms as shown in Figure 4(a). If the time constants do not match then there is a disparity in the measured waveforms as shown in Figure 4(b).

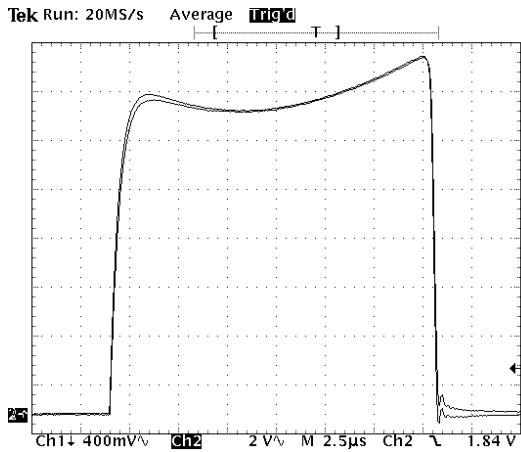
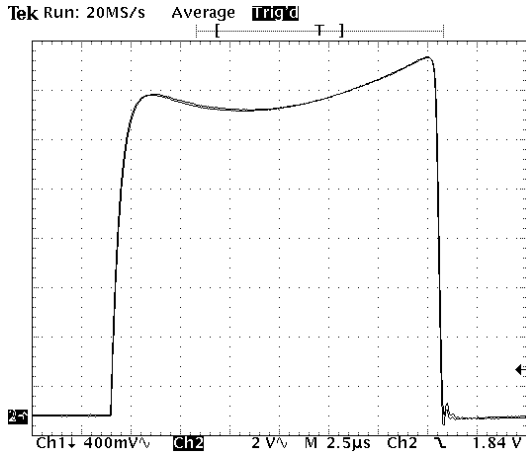


Figure 4. Comparison of a current pulse (200A/div) using a 2mV/A Rogowski transducer and a 20MHz CT

- (a) with correctly matched integrator time constants
- (b) with 3% mismatch of integrator time constants

For the 2mV/A transducer used for Figure 4(a), the phase error was checked at the transition frequency of 18kHz using a 306A pp sinusoidal current (8.5A in a 36 turn coil). The measurements from the Rogowski transducer (Ch1-X) and the CT (Ch2-Y) were viewed on an oscilloscope using an X-Y display trace. If there is a significant phase difference then the well known elliptical Lissajou figure is seen but if there is no phase difference the trace will appear as a straight line.

The phase difference  $\emptyset$  is given by

$$\emptyset = \sin^{-1} \left\{ \frac{\Delta X}{X_{pp}} \right\} = \sin^{-1} \left\{ \frac{\Delta Y}{Y_{pp}} \right\} \quad (8)$$

where  $X_{pp}$  is the peak-peak variation of the X signal and  $\Delta X$  is the figure width at  $Y=0$ .

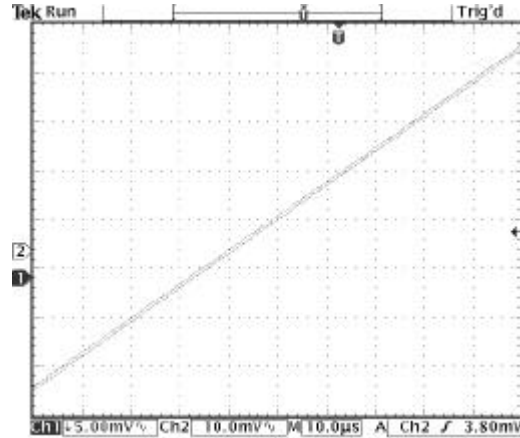


Figure 5. Expanded X-Y plot of sinusoidal current measurements using a 2mV/A Rogowski transducer and a 20MHz CT

Figure 5 shows the expanded X-Y trace with  $X_{pp} = 612mV$  for which the separation  $\Delta X$  can just be seen. The phase error is less than 0.1 degree.

#### 4. High Frequency Measurements

In order to create high frequency currents of sufficient magnitude for measurement using a 2mV/A CWT15 Rogowski transducer, a tuned circuit was used as shown in Figure 6. The 7 turn coil had a length of approx. 42mm and diameter of 13mm. All 7 turns linked the 500mm Rogowski coil and were positioned halfway between the free end and the fixed end – i.e. diametrically opposite the cable attachment as shown in Figure 7.

The tuning capacitor  $C_t$  was varied between  $0.47\mu F$  and  $0.5nF$  and for each value the signal generator frequency was adjusted to give resonance. The sinusoidal current measured by the CWT15 ranged from approximately 35App at 0.6MHz to 6.3App at 13MHz.

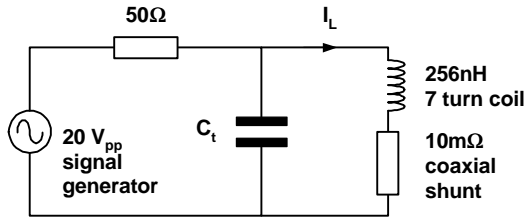


Figure 6. Tuned circuit for high frequency test currents



Figure 7. Photograph tuned circuit for high frequency test

The current was also measured using a 10mΩ 20MHz co-axial shunt which was used as the reference. Both the shunt and the CWT15 integrator were connected to an oscilloscope via 0.5m co-axial cables (the shunt connection having a 50 ohm termination) and the respective sinusoidal signals were compared both in magnitude and phase. The phase difference was measured using an x-y trace as described in section 3.

The measured values for the variation of CWT sensitivity and phase lag with frequency are compared with theoretical values in Table 1. The theoretical values are based on equation (3) with a 12.5ns cable delay, and estimated values based on coil and op-amp parameters for  $\hat{i}_c, T_c$  and  $T_b$  of 0.4525, 12.56ns and 10ns respectively. Correction  $F_1(\hat{\epsilon})$  and  $F_2(\hat{u})$  have then been applied and have an increasingly significant affect at frequencies above 6MHz.

The large phase lags are partly due to the 2.5m cable delay (eg.  $\hat{O}_a = 45$  degrees at 10MHz)

The agreement between the theoretical predictions and the measured values appears to be extremely good but the agreement may be largely fortuitous for three reasons.

Firstly the co-axial shunt has been taken to be a perfect reference whereas its sensitivity and phase are also function of high frequency and the measured results should be corrected to allow for the shunt dynamics. Tests on the shunt have shown that its sensitivity increases at very high frequency (i.e. it is slightly inductive) and therefore the CWT sensitivity values are likely to require slightly increasing.

Secondly the accuracy of measurement using an oscilloscope and with relatively small signals in the presence of noise could be as poor as 5% at these high frequencies.

Thirdly the theoretical representation of equation (3), as already stated, is itself an approximation.

FREQ	THEORETICAL VALUES				MEASURED VALUES	
	BEFORE CORRECTION		AFTER CORRECTION			
MHz	Mag. mV/A	Phase deg	Mag. mV/A	Phase deg	Mag. mV/A	Phase deg
0.635	2.00	-77	2.00	-7.4	2.01	-9.1
1.48	2.008	-18.1	2.01	-17.4	1.98	-18.3
2.06	2.014	-25.3	2.025	-24.3	1.99	-24.8
3.04	2.031	-37.5	2.05	-36.0	2.02	-35.9
6.48	2.125	-83.5	2.20	-80.5	2.22	-83.0
9.65	2.117	-133.4	2.27	-129.3	2.13	-135.6
13.32	1.600	-196.0	1.82	-191.2	1.82	-214.0

Table 1 Comparison of theoretical and measured values showing sensitivity variation with frequency.

Nevertheless the tests do indicate in general firstly reasonable agreement between theoretical prediction and actual performance and secondly that the 3dB bandwidth of a CWT15 Rogowski transducer is in excess of 10MHz.

A further interesting feature of the very high frequency performance is the variation in sensitivity due to current position. To test this the exciting coil shown in Figure 7 was positioned at 25% and 75% along the length of the coil from the free end as well as at 50% as for Table 1 above. This corresponds to  $\ddot{e} = 0.25$  and  $0.75$  in equation (4).

Magnitude measurements were taken at frequencies above 3MHz and compared with the  $\ddot{e} = 0.5$  measurement in order to give practical values for  $F_1(\ddot{e})$  relative to the  $\ddot{e} = 0.5$  value. Table 2 compares the theoretical and measured values

FREQ (MHz)	$\ddot{e} = 0.25$		$\ddot{e} = 0.5$	$\ddot{e} = 0.75$	
	Theor	Meas.	Ref.	Theor.	Meas.
3.04	1.010	1.018	1.003	0.992	0.991
6.48	1.047	1.056	1.014	0.960	0.946
9.65	1.115	1.096	1.031	0.905	0.900

Table 2 Variation of  $F_1(\ddot{e})$  with frequency and with current position  $\ddot{e}$ . ( $\ddot{e}$  is the p.u. current position from the coil free end with the current close to the coil edge).

Whereas the agreement between theory and measurement is only approximate, the results nevertheless illustrate the trend of the theoretical prediction, namely that as frequency increases the sensitivity increases towards the free end of the coil and decreases towards the fixed (or termination) end of the coil.

The theory also predicts that this is only a magnitude effect and the phase is independent of the current position  $\ddot{e}$ . This was confirmed by the practical measurements.

### Conclusion

The measurements taken confirm that the bandwidth for a CWT15 Rogowski transducer with a 500mm coil is at least 10MHz and that the theoretical model gives a reasonable

prediction of the behaviour up to this frequency.

The measurements also indicate that at frequencies approaching the bandwidth the transducer sensitivity is affected by the position of the current within the loop.

These results should be regarded as provisional and need to be confirmed using a better reference for comparison (i.e. a higher bandwidth CT or shunt) and preferably larger current signals. It is intended to report further on this when the paper is presented.

### References

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