# HIGH FREQUENCY EFFECTS IN CURRENT MEASUREMENT USING ROGOWSKI COILS

W F Ray, C R Hewson and J M Metcalfe
Power Electronic Measurements Ltd.
164 Lower Regent Street
Nottingham NG9 2DJ, England
Tel: +44/(0)115 9254 212
Fax: +44/(0)115 9677 685
E-Mail: info@pemuk.com

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## Keywords

Measurement, Transducer

## Abstract

Rogowski transducers are often used for measuring high frequency currents. It is not generally recognised that at very high frequencies the measurement is position sensitive. Furthermore the coil propagation time delay increases with frequency thereby reducing the expected bandwidth. This paper investigates these effects and presents some experimental verification.

## Introduction

Rogowski current transducers are frequently used for measuring very fast switching transients in power semiconductors, indeed they are often the only feasible method since high bandwidth current shunts or CTs cannot be introduced without affecting the behaviour of the switching circuit. Hence R & D engineers are often reliant on Rogowski measurements to certify the behaviour of their products without a second source of verification. The question naturally arises – how reliable are these measurements?

In practice the Rogowski coil is generally looped around the current carrying conductor (such as an IGBT lead) so that the current is in close proximity to part of the coil, usually mid way between the free end and the termination end as shown in Figure 1. In this case the voltage induced in the coil is concentrated over a relatively small number of turns and is not evenly distributed as in the symmetrical case where the current is central to the coil loop.

It is generally thought that, provided the coil is of uniform turns density and turn-area, then the overall voltage produced is independent of the current position within the coil loop. Whilst this is true at frequencies typically below 1MHz, at higher frequencies approaching the bandwidth of the coil the measurement is influenced by the current position. The variation is relatively small if the coil propagation angle $\theta = \omega . T_c < 0.5$ rad, where $T_c$ is the coil delay, but becomes more pronounced as $\theta$ increases.

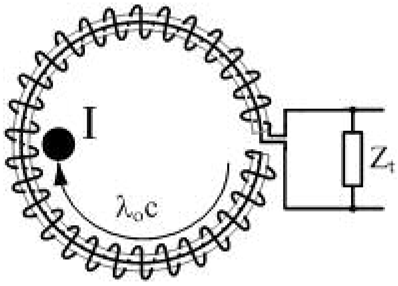
It is also generally thought that the coil may be represented by a transmission line model with uniformly distributed inductance and capacitance, as examined by Cooper [1] in his original paper on high frequency Rogowski transducer performance. Whilst providing the best method of analysing the hf performance, this model is only approximate at very high frequencies.

This aim of this paper is to examine the non-linear behaviour of Rogowski coils at high frequencies and to present some preliminary experimental measurements. The paper will not present a definitive analytical solution of hf behaviour since this is extremely complex but will help to identify the limitations of this method of current measurement.

## Transmission line model

A Rogowski coil has a helical winding, usually close packed, on a plastic former of zero conductivity and circular cross section. The free end of the winding is connected to the terminal end by an axial wire along the centre of the former so that connections can be made at the terminal end.

Figure 1. shows a Rogowski coil terminated with an impedance $Z_t$ and with a concentrated current **I** in close proximity to the coil at a position $\lambda_o$ relative to the free end – i.e. λ=0 represents the free end and λ=1 the termination end. Figure 2. shows the equivalent transmission line model with an induced voltage **ΔE** injected at a position λ. Bold symbols represent ac phasors.



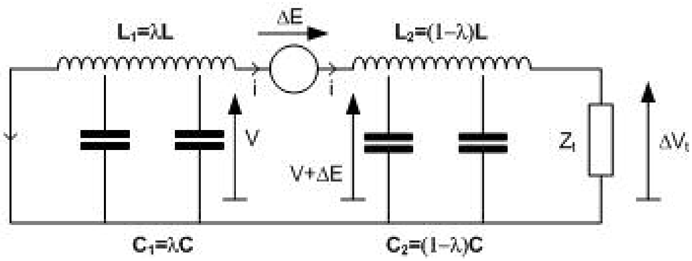




*Figure 1 Rogowski Coil.* *Figure 2 Transmission Line Model.*

Let the Rogowski coil have

| | | |
|---|---|---|
| Cross sectional area | A | $m^2$ |
| Turns density | N | turns/m |
| Circumference (length) | c | m |
| Distributed inductance | L | H |
| “ capacitance | C | F |
| “ resistance | R | Ω |

The former conductivity can be taken as zero.

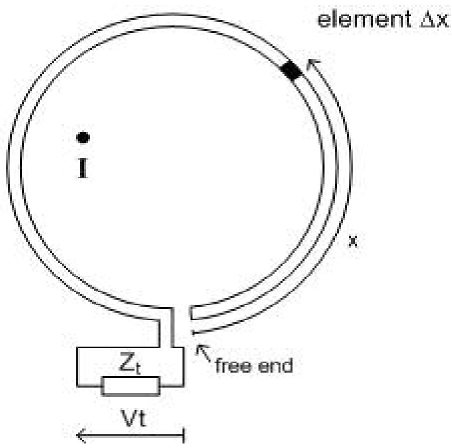



*Figure 3 Coil showing element Δx*

Consider an element of coil length Δx at distance x from the free end as shown in Figure 3, where x = λc. The current I can be at any position within the coil loop. Let the normal component of the magnetic field strength H due to I at position x be given by $\underline{H_n}$=F(x).I. (H is underlined to avoid confusion with the coil sensitivity which is given the symbol H (Vs/A)).

The flux linking 1 turn is $\mu_0$.A.F(x).I and the voltage ΔE induced in the element Δx is given by

$$\Delta E = N\Delta x.\mu_0 A.F(x)\frac{dI}{dt} \quad \text{or} \quad \frac{\Delta \boldsymbol{E}}{j\omega H_0 \boldsymbol{I}} = F(x).\Delta x \qquad (1)$$

where $H_0=\mu_0 N.A$ = coil sensitivity (Vs/A) at low frequency and **E** and **I** are phasors.

For Figure 2 let $L_1$=λ.L, $C_1$ = λ.C, $R_1$= λ.R, $L_2$=(1- λ).L, $C_1$==(1- λ).C, $R_1$==(1- λ).R.

For the free end part of the coil the standard transmission line equations give

$$\begin{bmatrix} \boldsymbol{V} \\ -\boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \cosh(\psi_1) & Z_1 \sinh(\psi_1) \\ Z_1^{-1} \sinh(\psi_1) & \cosh(\psi_1) \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{i_1} \end{bmatrix} \quad \text{where} \quad \begin{matrix} \psi_1 = \sqrt{(R_1 + j\omega L_1).j\omega C_1} \\ Z_1 = \sqrt{(R + j\omega L)/j\omega C} \end{matrix} \qquad (2)$$

Hence $\mathbf{V} = -Z_1.\tanh(\psi_1).\mathbf{i}$ (3)

For the termination end part of the coil

$$\begin{bmatrix} \boldsymbol{V + \Delta E} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \cosh(\psi_2) & Z_1 \sinh(\psi_2) \\ Z_1^{-1} \sinh(\psi_2) & \cosh(\psi_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta V_t} \\ \boldsymbol{\Delta V_t / Z_t} \end{bmatrix} \qquad (4)$$

where $\mathbf{\Delta V_t}$ is the contribution to the terminal voltage $\mathbf{V_t}$ due to the elemental voltage $\mathbf{\Delta E}$ induced in the coil element Δx.

Eliminating **V** and **I** in equations (3) and (4) gives

$$\frac{\boldsymbol{\Delta V_t}}{\boldsymbol{\Delta E}} = \frac{\cosh(\psi_1)}{\cosh(\psi) + (Z_1/Z_t).\sinh(\psi)} \quad \text{where} \quad \psi = \psi_1 + \psi_2 = \sqrt{(R + j\omega L).j\omega C} \qquad (5)$$

Substituting for **ΔE** from equation (1) and putting $\psi_1 = \lambda.\psi$ and x = λ.c gives

$$\frac{\boldsymbol{V_t}}{\boldsymbol{I}} = \frac{j\omega H_0}{\cosh(\psi) + (Z_1/Z_t).\sinh(\psi)} \int_0^1 c.\cosh(\lambda.\psi).F(\lambda).d\lambda \qquad (6)$$

For a circular coil loop of radius r with current I at the centre the magnetic field strength is the same for all λ and given by H = I/(2πr) = F(λ). The coil length c = 2πr. The integral is therefore sinh(λψ)/ψ and, as is already known [1],

$$\frac{\boldsymbol{V_t}}{j\omega H_0 \boldsymbol{I}} = \frac{\sinh(\psi)}{\psi.\{\cosh(\psi) + (Z_1/Z_t)\sinh(\psi)\}} \qquad (7)$$

## Theoretical Variation of Sensitivity with Current Position

Consider a circular coil loop as previously but with the current I situated a distance β.r from the loop centre as shown in figure (4). For the element r.Δφ the induced voltage ΔE is given by

$$\Delta E = \frac{d}{dt}\left\{ \mu_0 (N.r\Delta\phi) A \frac{I}{2\pi a} \cos(\alpha) \right\}$$

$$= \frac{1}{2\pi} H_0 \frac{dI}{dt} F(\phi).\Delta\phi$$

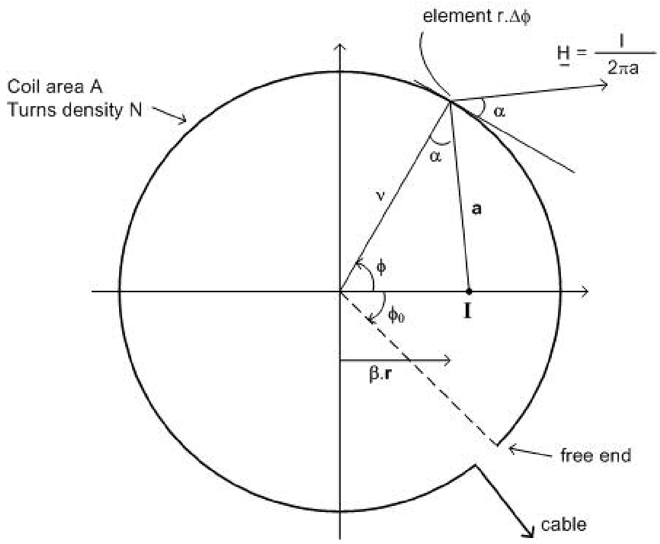



*Figure 4*

or $$\frac{\boldsymbol{\Delta E}}{j\omega H.\boldsymbol{I}} = \frac{1}{2\pi}F(\phi)\Delta\phi \qquad \text{where} \quad F(\phi) = \frac{r.\cos(\alpha)}{a} = \frac{1}{2}\left\{1 + \frac{r^2}{a^2}(1-\beta^2)\right\} \tag{8}$$

now $a^2 = (\beta r\text{-}r.\cos(\varphi))^2 + (r.\sin(\varphi))^2$ hence substituting for $a^2/r^2$ in equation (8) gives

$$F(\phi) = \frac{1}{2}\left\{1 + \frac{(1-\beta^2)}{2\beta}\frac{1}{k-\cos(\phi)}\right\} \qquad \text{where} \quad k = \frac{1}{2}\left\{\beta + \frac{1}{\beta}\right\} \geq 1 \tag{9}$$

Comparing equation (8) with (1), F(x).dx is replaced by $(1/2\pi).F(\varphi).\Delta\varphi$. Also $\lambda$ is replaced by $(\varphi+\varphi_0)/2\pi$. Hence utilising the result of (6)

$$\frac{\boldsymbol{V_t}}{\boldsymbol{I}} = \frac{j\omega H_0}{\cosh(\psi) + (Z_1/Z_t).\sinh(\psi)}\int_{-\phi_0}^{2\pi-\phi o} F_1(\phi).d\phi \tag{10}$$

where $$F_1(\phi) = \frac{1}{4\pi}\cosh\left\{\frac{\psi}{2\pi}(\phi+\phi_0)\right\}\left\{1 + \frac{(1-\beta^2)}{2\beta}\frac{1}{(k-\cos(\phi))}\right\}$$

$\int F_1(\varphi).d\varphi$ may be split into two parts to give

$$\int F_{11}(\phi).d\phi = \frac{1}{4\pi}\int_{-\phi_0}^{2\pi-\phi_0}\cosh\left\{\frac{\psi}{2\pi}(\phi+\phi_0)\right\}.d\phi = \frac{1}{2}\frac{\sinh(\psi)}{\psi} \tag{11}$$

$$\int F_{12}(\phi).d\phi = \frac{1-\beta^2}{8\pi\beta}\int_{-\phi_0}^{2\pi-\phi_0}\frac{\cosh\left\{\frac{\psi}{2\pi}(\phi+\phi_0)\right\}}{k-\cos(\phi)}.d\phi \tag{12}$$

There is no general analytic solution to integral (12). However for the limiting case of $k\rightarrow 1$ (i.e. $\beta\rightarrow 1$) the integrand $\rightarrow\infty$ at $\varphi = 0$. Hence the integral may be approximated by setting $\varphi = 0$ in the numerator giving

$$\int F_{12}(\phi).d\phi = \frac{1-\beta^2}{8\pi\beta}\cosh\left\{\frac{\phi_0}{2\pi}\psi\right\}\int_{-\phi_0}^{2\pi-\phi_0}\frac{1}{k-\cos(\phi)}.d\phi$$

It may be shown that for $0<\beta<1$ $$\int_{-\phi_0}^{2\pi-\phi_0}\frac{d\phi}{k-\cos(\phi)} = \frac{4\pi\beta}{1-\beta^2}$$

Hence, since $\lambda_0 = \varphi_0/(2\pi)$ $$\int F_{12}(\phi).d\phi = \frac{1}{2}\cosh(\lambda_0\psi)$$

Therefore, for a Rogowski coil with a current **I** inside the coil loop and close to the coil at position $\lambda_0$ as shown in Figure 1 the coil termination voltage $\mathbf{V_t}$ is related to **I** by

$$\frac{\boldsymbol{V_t}}{j\omega H_0\boldsymbol{I}} = \frac{0.5\left\{\frac{\sinh(\psi)}{\psi} + \cosh(\lambda_o\psi)\right\}}{\cosh(\psi) + \frac{Z_1}{Z_t}\sinh(\psi)} \tag{13}$$

Figure 5 shows the theoretical variation of coil sensitivity with current position $\lambda_o$ assuming that the coil resistance is negligible (i.e. $\psi = j\theta = j\omega\sqrt{(LC)}$ )and that the coil is terminated with $Z_t = R_t = \sqrt{(L/C)}$. Figure 5 also shows the variation for the symmetric excitation case which is very similar to the $\lambda$=0.5 case provided $\theta \leq \pi$. It will be seen that the variation of sensitivity is very significant for $\theta > \pi/2$.

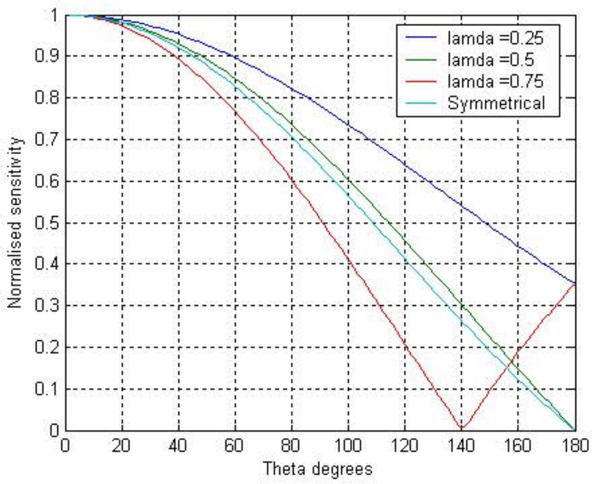



*Figure 5 Variation of Coil Sensitivity with Current Position $\lambda_0$ and coil angle $\theta=\omega\sqrt{(LC)}$.*

It may similarly be shown that for a Rogowski coil with a current **I** in the same direction as in Figure 1 but **outside** the coil loop and close to the coil at position $\lambda_0$, the coil termination voltage $\mathbf{V_t}$ is related to **I** by

$$\frac{\boldsymbol{V_t}}{j\omega H_0 \boldsymbol{I}} = \frac{0.5\left\{\dfrac{\sinh(\psi)}{\psi} - \cosh(\lambda_o\psi)\right\}}{\cosh(\psi) + \dfrac{Z_1}{Z_t}\sinh(\psi)} \tag{14}$$

In practice it is necessary to use a small, closely fitting, exciting coil as shown in Figure 6 for testing a Rogowski coil at high frequency since otherwise it is difficult to generate sufficient exciting current.

A single loop is equivalent to a current inside the Rogowski coil loop and a current in the opposite direction outside the loop. The relationship between the terminal voltage $\mathbf{V_t}$ and the exciting current **I**, where **I** is the ampere turns of the exciting coil, is obtained by subtracting equation (14) from (13).

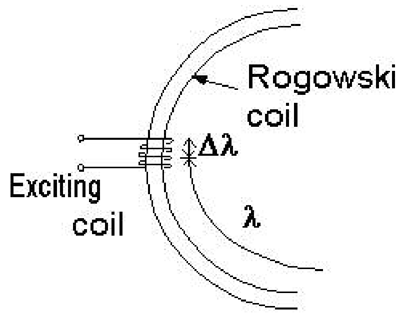



*Figure 6 Use of an exciting coil to test a Rogowski coil.*

Generally the exciting coil is distributed along a small proportion $\lambda_0 \pm \Delta\lambda$ of the Rogowski coil with an ampere turns density **I**/(2.Δλ). The current may therefore be represented by elemental contributions (**I**/(2.Δλ)).d$\lambda_o$. $\mathbf{V_t}$ is then given by summing the elemental voltage contributions. The resultant relationship between the terminal voltage $\mathbf{V_t}$ and the exciting current **I** (ampere turns) for Figure 6 is

$$\frac{\boldsymbol{V_t}}{j\omega H_0 \boldsymbol{I}} = \frac{\cosh(\lambda_0\psi)\left\{\dfrac{\sinh(\Delta\lambda.\psi)}{\Delta\lambda.\psi}\right\}}{\cosh(\psi) + \dfrac{Z_1}{Z_t}\sinh(\psi)} \tag{15}$$

If the excitation is evenly distributed along the entire coil (i.e. $\lambda_o$=0.5 and Δλ=0.5) then the numerator term becomes *sinh(Ψ)/Ψ*. This is the result obtained by Cooper [1] for a circular coil loop with a centrally situated current which also provides evenly distributed excitation.

# Implication of Sensitivity Variation at High Frequency

### For Coil Termination Impedance ≈ Coil Characteristic Impedance

This is usually the case for most applications for which at high frequency the integration is performed by a CR network with $R \approx \sqrt{(L/C)} >> (1/(\omega C))$.

Taking the $\lambda o$=0.5 case as typical the coil 3db bandwidth is typically the frequency for which $\theta = \pi/2$. For a typical 500mm coil with $T_c = \sqrt{(LC)}$ = 15ns this corresponds to a frequency of 16.6MHz. The bandwidth can be increased slightly by increasing the value of $R_t$ . For measuring current pulses or switching edges with little harmonic content above 10MHz this bandwidth will adequately reproduce the pulse shape. However for applications where it is required to measure sinusoidal currents of several MHz the current position within the loop should be taken into account.

### For Coil Termination Impedance << Coil Characteristic Impedance

For measuring very hf sinusoidal currents it is often best to utilise self integration [1,2,3] for which the coil is terminated with $R_t << \sqrt{(L/C)}$. This enables a higher bandwidth to be achieved for a given coil and has a much lower termination power loss compared with CR integration. Accurate symmetrical excitation is required to avoid resonances which will distort the measurement but this is difficult to achieve in practice. Even if the resonant frequencies are significantly higher than the measured current, any slight coil asymmetry and current harmonic content will excite these resonances.

# Experimental Results

It was decided to investigate resonant effects at high frequencies and to check if the theoretical predictions from the transmission line model were correct. A Rogowski coil with a relatively low natural frequency was selected so as to make the frequency response easier to measure. Tests were conducted on a 740mm closely-wound coil with a measured low frequency inductance of 240µH, capacitance 28pF, resistance 27Ω and sensitivity 72 nVs/A. The corresponding coil delay $T_c$ = 82ns and natural frequency 1.94 MHz.

The coil was excited with a small 10 turn coil as shown in Figure 6.and was terminated with 50Ω at the oscilloscope input. This gave $Z_1/Z_t \approx 60$ which was sufficiently high to show the resonances whilst avoiding connection problems between coil and scope.

### Test current at mid coil position ( $\lambda_o$=0.5 )

Consider the case with the current close to the coil and assume the coil resistance is negligible – i.e. $\psi = j\theta$. From equation (15), neglecting the small $\Delta\theta$ effect

$$\frac{\boldsymbol{V}_t}{j\omega H_0 \boldsymbol{I}} = \frac{\cos(\lambda_0\theta)}{\cos(\theta) + j\dfrac{Z_1}{Z_t}\sin(\theta)} \qquad (16)$$

For $Z_1 >> Z_t$ and $\lambda_o$=0.5 the resonant frequencies are expected to occur at $\theta \approx 2\pi, 4\pi, 6\pi$ etc., i.e. at frequencies of 12.2, 24.4, 36.6 ….MHz.

The coil was excited over the frequency range 1 to 40 MHz and the coil sensitivity $H(f)=\mathbf{V_t}/(j\omega\mathbf{I})$ was measured. The coil resistance was estimated to increase at high frequency due to

skin and proximity effects according to the relationship $R=120.\log_e(f(Mhz))$. Using eqn (15) with $\Delta\lambda=0.06$, theoretical values for H(f) were calculated.
Figure 7 shows the comparison of measured and theoretical values of coil sensitivity H for constant inductance. It will be seen that the measured resonances occurred at frequencies lower than predicted from the expected value of $T_c$ (determined by separately measuring L and C for the coil at a relatively low frequency - typically 10kHz). It would appear that the LC product increases at high frequency.

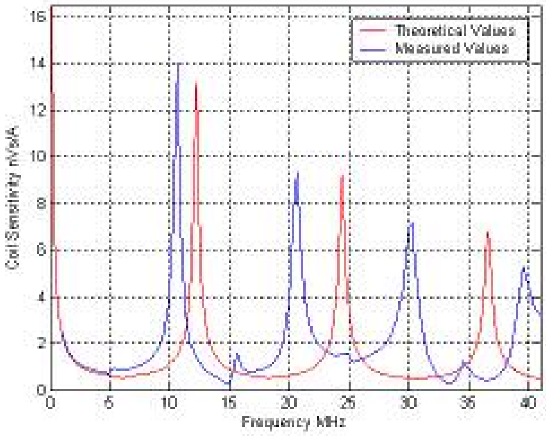



*Figure 7 Variation of coil sensitivity with frequency for constant inductance.*

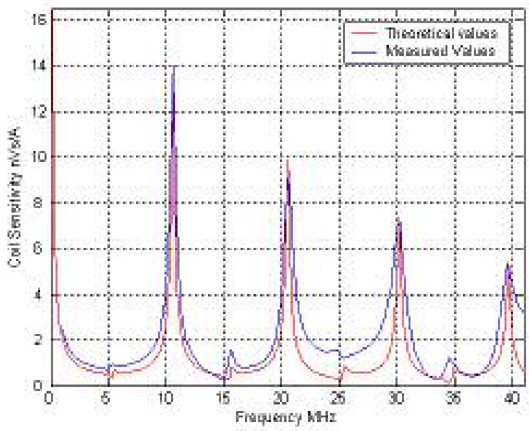



*Figure 8 Variation of coil sensitivity with frequency for increasing inductance.*

## Increase of $T_C = \sqrt{(LC)}$ at High Frequency

Preliminary electromagnetic modelling of a helical closely wound coil at Nottingham University has verified that this effect is present. At very high frequencies the coil transmission line model is only approximate and the coil has to be treated in similar manner to a waveguide. Nevertheless for sinusoidal excitation at a particular frequency it is thought that the behaviour can be modelled by a transmission line provided the inductance increases with frequency.

For each measured resonant frequency $f_R$ the corresponding value of $T_c$ could be calculated as $T_c = n / f_R$ where n =1,2,3…. is the resonance number. These values agreed well with the electromagnetic model as shown in Table 1 below. To fit the measured values it was found that L had to be increased by the empirical relationship $L = L_0.(1+(f/280)^{0..34})$ where f is the frequency in MHz and $L_0$ is the low frequency value of 240µH.

| Resonance | | Coil delay $T_c$ (ns) | | |
|---|---|---|---|---|
| n | $f_R$ (MHz) | Calculated $= n / f_R$ | Predicted by electro-magnetic model | Calculated from $L = L_0.(1+(f/280)^{0..34})$ |
| 1 | 10.6 | 94.3 | 95.0 | 94.5 |
| 2 | 20.6 | 97.1 | 97.3 | 97.4 |
| 3 | 30.3 | 99.0 | 98.9 | 99.4 |
| 4 | 39.3 | 101.8 | 100.3 | 100.9 |

Table 1 Comparison of measured and calculated values for $T_c$

Figure 8 shows the comparison of measured and theoretical values of coil sensitivity H as for figure 6 but with increasing inductance according to the empirical relationship given above. It will be seen that not only are the resonant frequencies in agreement but there is also reasonable agreement between these resonances at least as the shape of the response is concerned. For the low sensitivities between the resonances the experimental accuracy was less.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The results vindicate the relationship given by equation (15) at least for the case of $\lambda_0 = 0.5$. It should be noted that without the $\cosh(\lambda_0\psi)$ in the numerator of (15) there would be resonances at $\theta = n\pi$ for odd value of integer n as well as even values. These other resonances are clearly missing.

### Test current at other coil positions ( $\lambda_o$=0.25 and 0.75 )

The measurements were repeated at other positions of the exciting coil, 25% and 75% of the coil length from the free end. To make the measured and theoretical resonant frequencies agree it was necessary to amend the empirical law for increasing inductance to $L = L_0.(1+(f/130)^{0..34})$, so that the increase is higher than for the mid position case. Figures 9 and 10 show the comparisons of theoretical and measured values of coil sensitivity for varying frequency.

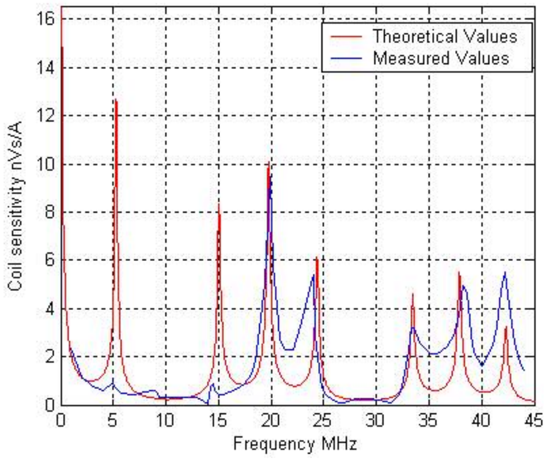


*Figure 9 Variation of coil sensitivity with frequency for increasing inductance $\lambda_0$=0.25*

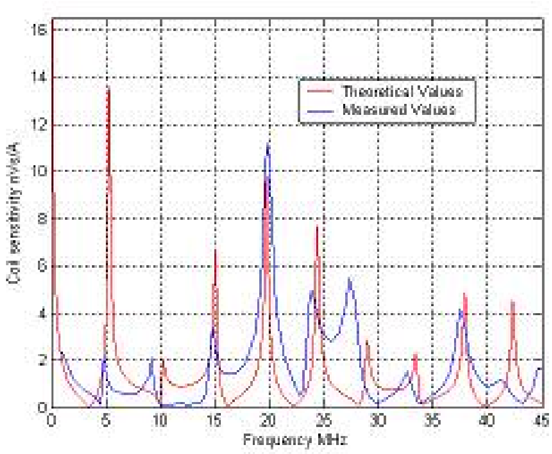


*Figure 10 Variation of coil sensitivity with frequency for increasing inductance $\lambda_0$=0.75*

For both $\lambda_o$=0.25 and $\lambda_o$=0.75 equation (16) predicts resonances at $\theta \approx n\pi$ where n = 1, 3, 4, 5, 7,8 etc. i.e. there should be no resonances for n = 2, 6, 10. This is seen to be the case.

For frequencies from 5 to 15 MHz the measured waveform was very distorted which may explain why the expected resonances at 5 and 15 MHz were not detected. The reason is not known at present - at these high frequencies there are significant electric and magnetic fields outside the coil due to its propagation and these are affected by the proximity of the person making the measurements. Also the frequency response changes significantly with change in $\lambda_0$ and the actual values may not be exactly 0.25 and 0.75.

Hence it is not altogether surprising that the correlation between measurement and theory is not very exact for Figures 9 and 10. Nevertheless the theory does predict the general shape of the frequency response.

It must be remembered that these frequencies are significantly higher than the coil bandwidth and that, as previously stated, it is not recommended to use the coil for measurement at coil angles greater than $\pi/2$ which corresponds to approx 3 MHz for the coil under test.

## Conclusions

The behaviour of a Rogowski coil for measuring a currents at high frequency has been investigated, in particular a theoretical relationship between the coil terminal voltage and a current close to the coil surface has been derived. This relationship has predicted that for coil angles above 30 deg (or frequency > $1/(12T_c)$ the coil sensitivity becomes increasingly dependant on the position of the current within the coil loop.

The relationship also predicts that if the coil termination impedance is much less than its characteristic impedance, resonances will occur at higher frequencies $f_R = n / (2T_c)$ but that, depending on current position, resonances for some values of integer n may be absent (in particular for the current in the coil mid position only even values of n give resonances).

It has also been found that the transmission line model of the coil is inaccurate at high frequencies although it can be approximated by increasing the coil inductance with frequency. Further electromagnetic modelling is required to establish a theoretical relationship between equivalent inductance and frequency (if this can be done). Perhaps only a full electromagnetic model will provide accurate predictions.

There was good agreement between theoretical predictions and measured values for the case of excitation at the coil mid position but the agreement was poor for excitation at positions 25% and 75% along the coil length. Further experimental results are required before a definitive conclusion can be made. Nevertheless the theory did predict the resonant frequencies and the general shape of the response.

How do these findings affect the general use of Rogowski transducers for measuring current waveforms or pulses ? The answer is not much since generally the transducer bandwidth is below the frequency at which these positional effects commence and generally the coil is properly terminated so that resonances do not occur.

However in the few applications where Lr (self) integration is used (usually only for measuring sinusoidal current waveforms) resonances can affect the measurement unless the resonant frequencies are much higher than the measured current and hence it is useful to predict these.

From the results of this paper, for best accuracy at high frequencies approaching the transducer bandwidth, the Rogowski coil should be looped around the current carrying conductor such that the conductor position lies between the centre of the coil loop and the point half way along the length of the coil.

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